

Different Integrals (classification)

Let us consider a partial differential equation of order one as

$$f(x, y, z, p, q) = 0 \quad \text{--- (1)}$$

Let its solⁿ be $F(x, y, z, a, b) = 0$ --- (2)

having equal number of constants a, b as many an independent variable x, y .

Complete integrals: \rightarrow Here eqⁿ (2) is called complete integral of (1)

Particular integrals: \rightarrow Particular values to 'a' & 'b' in eqⁿ (2) we get particular solution (Particular integral)

Singular integral: \rightarrow Here eqⁿ (2)

i.e. $F(x, y, z, a, b) = 0$ is complete integral.

Differentiating (2) by a we get

$$\frac{\partial F}{\partial a} = 0 \quad \text{--- (3)}$$

Differentiating (2) by b we get

$$\frac{\partial F}{\partial b} = 0 \quad \text{--- (4)}$$

eliminating a, b from (2), (3), (4) we get singular solⁿ.

General integrals \rightarrow Complete integral is

$$F(x, y, z, a, b) = 0 \quad \text{--- (2)}$$

Let us put $b = \phi(a)$ then (2) is

$$F(x, y, z, a, \phi(a)) = 0 \quad \text{--- (5)}$$

Differentiating (5) w.r. to a , we get

$$F'(x, y, z, \phi'(a)) = 0 \quad \text{--- (6)}$$

Eliminating 'a' from (3), (6) of

$b = \phi(a)$ we get the general solution.